Callable Bond and Valuation

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https://finpricing.com/lib/IrCurve.html
Callable Bond

Summary

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- The Advantages of Callable Bonds
- Callable Bond Payoffs
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Callable Bond

Callable Bond Definition

◆ A callable bond is a bond in which the issuer has the right to call the bond at specified times (callable dates) from the investor for a specified price (call price).

◆ At each callable date prior to the bond maturity, the issuer may recall the bond from its investor by returning the investor’s money.

◆ The underlying bond can be a fixed rate bond or a floating rate bond.

◆ A callable bond can therefore be considered a vanilla underlying bond with an embedded Bermudan style option.

◆ Callable bonds protect issuers. Therefore, a callable bond normally pays the investor a higher coupon than a non-callable bond.
Callable bond

Advantages of Callable Bond

- Although a callable bond is a higher cost to the issuer and an uncertainty to the investor comparing to a regular bond, it is actually quite attractive to both issuers and investors.
- For issuers, callable bonds allow them to reduce interest costs at a future date should rate decrease.
- For investors, callable bonds allow them to earn a higher interest rate of return until the bonds are called off.
- If interest rates have declined since the issuer first issues the bond, the issuer is likely to call its current bond and reissues it at a lower coupon.
Callable Bond Payoffs

◆ At the bond maturity $T$, the payoff of a callable bond is given by

$$V_c(t) = \begin{cases} F + C & \text{if not called} \\ \min(P_c, F + C) & \text{if called} \end{cases}$$

where $F$ – the principal or face value; $C$ – the coupon; $P_c$ – the call price; $\min(x, y)$ – the minimum of $x$ and $y$

◆ The payoff of the callable bond at any call date $T_i$ can be expressed as

$$V_c(T_i) = \begin{cases} \overline{V}_{T_i} & \text{if not called} \\ \min(P_c, \overline{V}_{T_i}) & \text{if called} \end{cases}$$

where $\overline{V}_{T_i}$ – continuation value at $T_i$
Callable Bond

Model Selection Criteria

- Given the valuation complexity of callable bonds, there is no closed form solution. Therefore, we need to select an interest rate term structure model and a numerical solution to price them numerically.

- The selection of interest rate term structure models
  - Popular interest rate term structure models:
    - Hull-White, Linear Gaussian Model (LGM), Quadratic Gaussian Model (QGM), Heath Jarrow Morton (HJM), Libor Market Model (LMM).
    - HJM and LMM are too complex.
    - Hull-White is inaccurate for computing sensitivities.
    - Therefore, we choose either LGM or QGM.
Callable Bond

Model Selection Criteria (Cont)

◆ The selection of numeric approaches
  ◆ After selecting a term structure model, we need to choose a numerical approach to approximate the underlying stochastic process of the model.
  ◆ Commonly used numeric approaches are tree, partial differential equation (PDE), lattice and Monte Carlo simulation.
  ◆ Tree and Monte Carlo are notorious for inaccuracy on sensitivity calculation.
  ◆ Therefore, we choose either PDE or lattice.
  ◆ Our decision is to use LGM plus lattice.
The dynamics
\[ dX(t) = \alpha(t)dW \]
where \( X \) is the single state variable and \( W \) is the Wiener process.

The numeraire is given by
\[ N(t, X) = \left( H(t)X + 0.5H^2(t)\zeta(t) \right)/D(t) \]

The zero coupon bond price is
\[ B(t, X; T) = D(T)\exp\left( -H(t)X - 0.5H^2(t)\zeta(t) \right) \]
Callable Bond

LGM Assumption

◆ The LGM model is mathematically equivalent to the Hull-White model but offers
  ◆ Significant improvement of stability and accuracy for calibration.
  ◆ Significant improvement of stability and accuracy for sensitivity calculation.
◆ The state variable is normally distributed under the appropriate measure.
◆ The LGM model has only one stochastic driver (one-factor), thus changes in rates are perfectly correlated.
Callable Bond

LGM calibration

- Match today’s curve
  At time $t=0$, $X(0)=0$ and $H(0)=0$. Thus $Z(0,0;T)=D(T)$. In other words, the LGM automatically fits today’s discount curve.

- Select a group of market swaptions.

- Solve parameters by minimizing the relative error between the market swaption prices and the LGM model swaption prices.
Callable Bond

Valuation Implementation

- Calibrate the LGM model.
- Create the lattice based on the LGM: the grid range should cover at least 3 standard deviations.
- Calculate the payoff of the callable bond at each final note.
- Conduct backward induction process iteratively rolling back from final dates until reaching the valuation date.
- Compare exercise values with intrinsic values at each exercise date.
- The value at the valuation date is the price of the callable bond.
## Callabe Bond

### A real world example

<table>
<thead>
<tr>
<th>Bond specification</th>
<th>Callable schedule</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Buy Sell</strong></td>
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<td><strong>Interest Accrual Date</strong></td>
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<td>1/30/2013</td>
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<tr>
<td><strong>Last Coupon Date</strong></td>
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<tr>
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</table>
Thanks!

You can find more details at
https://finpricing.com/lib/FiCallableBond.html